

a) diagonaldominant
 \hookrightarrow konvergiert

b)

$$L = \begin{pmatrix} 0 & 0 & 0 \\ 5 & 0 & 0 \\ 4 & 2 & 0 \end{pmatrix} \quad D = \begin{pmatrix} 8 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 7 \end{pmatrix}$$

$$R = \begin{pmatrix} 0 & 5 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$(D + L)^{-1} = \begin{pmatrix} 8 & 0 & 0 \\ 5 & 9 & 0 \\ 4 & 2 & 7 \end{pmatrix}^{-1} = \begin{pmatrix} -0,125 & 0 & 0 \\ 0,069 & -0,111 & 0 \\ 0,051 & 0,032 & -0,143 \end{pmatrix}$$

$$X^{(k+1)} = \begin{pmatrix} 0 & -0,625 & -0,25 \\ 0 & 0,347 & 0,028 \\ 0 & 0,258 & 0,135 \end{pmatrix} X^{(k)} + \begin{pmatrix} 2,375 \\ -0,764 \\ 3,718 \end{pmatrix}$$

$$X^0 = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} \quad X^1 = \begin{pmatrix} 2,25 \\ -1,028 \\ 3,865 \end{pmatrix} \quad X^2 = \begin{pmatrix} 2,051 \\ -1,013 \\ 3,975 \end{pmatrix}$$

$$X^3 = \begin{pmatrix} 2,015 \\ -1,055 \\ 3,99 \end{pmatrix}$$

$$c) \quad B = \begin{pmatrix} 0 & -0,625 & -0,25 \\ 0 & 0,347 & 0,028 \\ 0 & 0,258 & 0,135 \end{pmatrix}$$

$$\|B\|_{\infty} = 0,875$$

$$\begin{aligned} \|x^3 - \bar{x}\| &\leq \frac{0,875}{0,125} \left\| \begin{pmatrix} 2,015 \\ -1,055 \\ 3,99 \end{pmatrix} - \begin{pmatrix} 2,051 \\ -1,013 \\ 3,975 \end{pmatrix} \right\|_{\infty} \\ &\leq 7 \cdot \left\| \begin{pmatrix} -0,036 \\ 0,008 \\ 0,018 \end{pmatrix} \right\|_{\infty} \end{aligned}$$

$$\begin{aligned} \|x^3 - \bar{x}\| &\leq 7 \cdot 0,036378 \\ &= 0,255 \end{aligned}$$

$$d) \quad 0,0001 \leq \frac{0,875^n}{0,125} \left\| \begin{pmatrix} 2,25 \\ -1,028 \\ 3,865 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} \right\|_{\infty}$$

$$0,0001 \leq \frac{0,875^n}{0,125} - 1,25$$

$$n \geq \frac{\log((0,0001 \cdot 0,125) + 1,25)}{\log(0,875)} = 86,2$$

$$e) \quad 0,0001 \leq \frac{0,875^n}{0,125} \left\| \begin{pmatrix} 2,015 \\ -1,055 \\ 3,39 \end{pmatrix} - \begin{pmatrix} 2,051 \\ -1,013 \\ 3,375 \end{pmatrix} \right\|_{\infty}$$

$$0,0001 \leq \frac{0,875^n}{0,125} \cdot 0,0364$$

$$n \geq \frac{\log((0,0001 \cdot 0,125); 0,0364)}{\log(0,875)}$$

$$= 59,7$$